

Time: 2 Hours

Marks: 60

- Note : 1) Question no 1 is compulsory  
 2) Attempt any three questions from the remaining.  
 3) Assume suitable data, if required and state it clearly.

- Q.1 Solve any FIVE questions (15 M)
- a) Find  $L[e^{-2t} \cdot \cos^2 t]$  (03 M)
- b) Find the Fourier Series of  $f(x) = x$  in the interval  $(-1,1)$  (03 M)
- c) Find the Fourier transform of signal  $X(t)$  where  $X(t) = e^{-at}u(t)$  where  $a > 0$ . (03 M)
- d) Find Z- transform of  $f(k) = a^k, k \geq 0$  (03 M)
- e) The probability mass function of a random variable X is (03 M)
- |            |   |    |   |    |    |
|------------|---|----|---|----|----|
| $X = x$    | 0 | 1  | 2 | 3  | 4  |
| $P(X = x)$ | K | 2k | k | 4k | 3k |
- Find  $k$  and  $P(X \leq 2)$
- f) Three of the eigen values of  $5 \times 5$  matrix are 1,2 & 3. If the determinant and trace of matrix are 24 & 11 respectively, find the remaining Eigen values. (03 M)

- Q.2 Solve all questions: (15 M)
- a) Obtain  $Z\{1\}$  and hence prove that  $Z\{k\} = \frac{z}{(z-1)^2}, |z| > 1$  (04 M)
- b) The local authorities in a certain city installed 10,000 electric lamps in the streets of the city these lamps have average life of 1000 burning hours with a standard deviation of 200 hours. What number of lamps might be expected to fail in first 800 hours. (05 M)
- c) Find the eigen values and eigen vectors of the matrix  $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$  (06 M)

- Q.3 Solve all questions: (15 M)
- a) Find Laplace transform of  $\frac{1-\cos t}{t}$  (04 M)
- b) Find the Fourier transform of  $X(t) = \cos(\omega t)$  (05 M)
- c) Obtain the Fourier expansion of  $f(x) = (x - 2\pi)^2$  in the interval  $0 \leq x \leq 2\pi$  (06 M)

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Q.4 Solve all questions: (15 M)

a) If  $f(k) = \frac{1}{3^k}$ ,  $k \geq 0$  and  $g(k) = \frac{1}{5^k}$ ,  $k \geq 0$  then find Z- transform of  $f(k) * g(k)$  (04 M)

b) Check whether matrix  $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$  is diagonalizable or not? If so then find the diagonal form and transforming form. (05 M)

c) If the pdf of a random variable is given by (06 M)

$$f(x) = kx, \quad 0 \leq x \leq 2$$

$$= 6k - kx, \quad 2 \leq x \leq 4$$

Find (i)  $k$ , (ii)  $E(X)$ , (iii)  $V(X)$

Q.5 Solve all questions: (15 M)

a) Find the inverse Z-transform of  $F(z) = \frac{1}{(z-3)(z-2)}$  if ROC is  $|z| < 2$  (04 M)

b) Find  $L^{-1} \left[ \frac{s}{(s-1)^2(s-2)} \right]$  using partial fraction. (05 M)

c) Let  $X_1(t)$  &  $X_2(t)$  be two signals defined by (06 M)

$$X_1(t) = te^{-t}u(t) \text{ \& } X_2(t) = te^{-t}u(t).$$

Find Fourier Transform of the convolution  $X_1(t) * X_2(t)$ .

Q.6 Solve all questions: (15 M)

a) Let  $X(t)$  be a signal defined as  $X(t) = \begin{cases} 1 - t^2, & |t| < 1 \\ 0, & |t| > 1 \end{cases}$ . Find the Fourier transform of  $X(t)$ . (04 M)

b) Obtain complex form of Fourier series for  $f(x) = \cosh 3x + \sinh 3x$  in  $(-3,3)$  (05 M)

c) Solve using Laplace transform  $L \frac{di}{dt} + Ri = Ee^{-at}$  where  $i(0) = 0$ . (06 M)

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